Math 2310, Linear Algebra

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Final Exam Review Worksheet

Work out each problem. When you finish, on the last page find the blank space for each question number and fill in its corresponding letter. Question number 0 is done as an example.

0. If the dot product of two vectors is zero, the two vectors are ___________________.

(T) eigenvectors (Y) orthogonal (P) null vectors (L) Yo mama

(1) Find the value(s) of h that make the augmented matrix a *consistent* linear system: $\begin{bmatrix} 1 & h & 4 \\ -3 & 6 & 9 \end{bmatrix}$.

(A) $h = 6$ (I) $h = -2$ (O) $h \neq -2$ (E) $h \neq 6$

(2) Describe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric vector form for $A = \begin{bmatrix} 1 & 3 & 0 & -4 \\ 2 & 6 & 0 & -8 \end{bmatrix}$.

(U) $\text{Span}\{\begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 0 \\ 1 \end{bmatrix}\}$ (P) $\text{Span}\{\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \end{bmatrix}, \begin{bmatrix} -4 \\ -8 \end{bmatrix}\}$

(M) $\text{Span}\{\begin{bmatrix} -3 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \end{bmatrix}\}$ (S) $\text{Span}\{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}\}$

Determine by inspection if the following 5 sets are linearly dependent or linearly independent. If you cannot figure it out by inspection then find out some other way. Explain your reasoning.

(3) $\left\{\begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 9 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix}\right\}$, (L) linearly dependent (B) linearly independent

(4) $\left\{\begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 9 \end{bmatrix}, \begin{bmatrix} 2 \\ -28 \\ 3 \end{bmatrix}\right\}$, (K) linearly dependent (M) linearly independent

(5) $\left\{\begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 9 \end{bmatrix}\right\}$, (R) linearly dependent (J) linearly independent

(6) $\left\{ \begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -28 \\ 3 \end{bmatrix} \right\}$, (S) linearly dependent (L) linearly independent

(7) $\left\{ \begin{bmatrix} -2 \\ 4 \\ 6 \\ -10 \end{bmatrix}, \begin{bmatrix} 3 \\ -6 \\ -9 \\ 15 \end{bmatrix} \right\}$, (T) linearly dependent (O) linearly independent

(8) Consider the linear transformation $T(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}) = \begin{bmatrix} 0 \\ x_1 + x_2 \\ x_2 + x_3 \\ x_3 + x_4 \end{bmatrix}$. Find a matrix A that implements the mapping $T(\mathbf{x}) = A\mathbf{x}$, and then state if the transformation is one-to-one or onto (or both or neither).

(A) One-to-one (O) onto (E) both (I) neither

(9) Find the inverse of $\begin{bmatrix} 1 & 5 \\ 5 & 11 \end{bmatrix}$, if it exists.

(E) $\begin{bmatrix} -11/14 & 5/14 \\ 5/14 & -1/14 \end{bmatrix}$ (S) $\begin{bmatrix} 11 & -5 \\ -5 & 1 \end{bmatrix}$ (A) $\frac{1}{14}\begin{bmatrix} 11 & -5 \\ -5 & 1 \end{bmatrix}$ (T) $\begin{bmatrix} -5 & 11 \\ 1 & -5 \end{bmatrix}$

(10) Calculate det $\begin{bmatrix} 4 & 0 & 0 & 4 \\ 7 & -1 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 5 & -8 & 4 & -3 \end{bmatrix}$.

(M) 36 (A) -36 (R) -1280 (S) 0

(11) Let $A = \begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$. Is $\mathbf{w}$ is Col A? Is $\mathbf{w}$ in Nul A?

(R) In Col A only (A) In Nul A only (P) both (T) neither

(12) Let $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ be a set that spans $\mathbb{R}^p$. Is S a basis for $\mathbb{R}^p$?

(F) Yes (N) No

(13) Let S be the following set of vectors in $\mathbb{P}_3 : \{1+t^3, 3+t-2t^2, -t+3t^2-t^3\}$. Use coordinate vectors to find out if the vectors in S are linearly independent. Does S span $\mathbb{P}_3$? Is S a basis?

(S) Linearly independent, does span, is a basis

(P) Linearly independent, does span, is not a basis

(A) Not linearly independent, does not span, is not a basis

(C) Linearly independent, does not span, is not a basis

(E) Not linearly independent, does span, is not a basis

(14) Let A be a 5×6 matrix. What is the largest possible rank of A? What is the largest possible dimension for the null space of A? What is the smallest possible dimension for Row A?

(P) 6, 6, 1 (R) 5, 6, 1 (O) 5, 6, 0 (D) 6, 5, 0

(15) Find the characteristic polynomial and eigenvalues of $A = \begin{bmatrix} 1 & -5 \\ 5 & 2 \end{bmatrix}$.

(R) $\lambda^2 - 3\lambda - 23$, e.values are 1, 2 (U) $\lambda^2 - 3\lambda + 27$, no real eigenvalues
(M) $\lambda^2 - 3\lambda + 27$, e.values are 1, 2 (I) $\lambda^3 - 5\lambda^2 + 5\lambda + 2$, e.values are 1, 2, 5, -5

(16) The matrix $A = \begin{bmatrix} 13 & -4 \\ 42 & -13 \end{bmatrix}$ has eigenvalues 1 and -1. Diagonalize A and use the diagonalization to calculate A^{200}.

(I) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (R) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (A) $\begin{bmatrix} 13 & -4 \\ 42 & -13 \end{bmatrix}$ (T) $\begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$

(17) Let $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$. Is $A\mathbf{x} = \mathbf{b}$ consistent? If not, find a least-squares solution of $A\mathbf{x} = \mathbf{b}$.

(T) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (R) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (E) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (O) $\begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}$

Fill in each of the letters below to find out what the man is saying to the (cat)woman.

$\overline{0}$ $\overline{1}$ $\overline{2}$ $\overline{3}$ $\overline{14}$ $\overline{1}$ $\overline{4}$ $\overline{5}$ $\overline{15}$ $\overline{6}$ $\overline{7}$

$\overline{3}$ $\overline{8}$ $\overline{4}$ $\overline{9}$ $\overline{0}$ $\overline{14}$ $\overline{2}$ $\overline{10}$

$\overline{11}$ $\overline{10}$ $\overline{1}$ $\overline{12}$ $\overline{16}$ $\overline{3}$ $\overline{17}$ $\overline{11}$ $\overline{8}$ $\overline{13}$ $\overline{7}$ $\overline{15}$ $\overline{10}$ $\overline{9}$